4.2.2

By definition its arclength is given by

$$\int_0^1 \|(0, 6t, 3t^2)\| = \int_0^1 \sqrt{36t^2 + 9t^4} = \int_0^1 3t\sqrt{t^2+4} = \left[(t^2+4)^{3/2}\right]_0^1 = 5\sqrt{5} - 8$$

4.16.2

(a) Since $T(t) \cdot T(t) = 1$, we have after differentiating that $2T(t) \cdot T'(t) = 0$, which gives the desired result.

(b) We have $T(t) = c'(t)/\|c'(t)\| = c'(t)(c'(t) \cdot c'(t))^{-1/2}$.

Assuming that

$$\frac{\partial (c'(t) \cdot c'(t))^{-1/2}}{\partial t} = (c(t) \cdot c'(t))(c'(t) \cdot c'(t))^{-3/2} = \frac{c(t) \cdot c'(t)}{\|c'(t)\|^3}$$

We get finally that

$$T'(t) = \frac{c''(t)}{\|c'(t)\|} + \frac{c'(t)(c(t) \cdot c'(t))}{\|c'(t)\|^3}$$

Another approach to this problem would be to write $c(t) = (x(t), y(t), z(t))$ and do the corresponding calculations.

5.2.3

(a)

$$\int_{-1}^1 \int_0^1 (x^4 y + y^2) dy\ dx = \int_{-1}^1 \left(\frac{x^4}{2} + \frac{1}{3}\right) dx = \frac{13}{15}$$

(b)

$$\int_0^{\pi/2} \int_0^1 (y \cos x + 2) dy\ dx = \int_0^{\pi/2} \left(\frac{\cos x}{2} + 2\right) dx = \frac{1}{2} + \pi$$

(c) Taking into account that the primitive of xe^x is $xe^x - e^x$ we have that

$$\int_0^1 \int_0^1 (xe^x)(ye^y) dy\ dx = \int_0^1 (xe^x) dx = 1$$

(d) Now assuming that the primitive of $\log y$ is $y \log y - y$, we have

$$\int_{-1}^{0} \int_{1}^{2} (-x \log y) dy\ dx = \int_{-1}^{0} -x(2 \log 2 - 1) dx = \log 2 - \frac{1}{2}$$

5.1.6

By Cavalieri's Principle, the volume is given by

$$V = \int_{0}^{7} A(h) dh$$

Where $A(h)$ is the area of the section of the figure at height h. Since this section is always a rectangle of dimensions 5×3, we have that $A(h) = 15$ for all h, then

$$V = \int_{0}^{7} 15\ dh = 105$$

5.1.10

Note that y is negative at all points of the rectangle, therefore we can substitute $|y|$ by $-y$.

Then we have

$$\int_{0}^{-2} \int_{-1}^{0} \left(-y \cos \frac{1}{4}\pi x\right) dy\ dx = \int_{0}^{-2} \left(\frac{1}{2} \cos \frac{1}{4}\pi x\right) dx = \left[\frac{4}{2\pi} \sin \frac{\pi x}{2}\right]_{0}^{2} = \frac{2}{\pi}$$

5.2.8

The region is bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = x^2 + y^4$. This means that

$$V = \int_{0}^{1} \int_{0}^{1} \int_{0}^{x^2+y^4} dz\ dy\ dx = \int_{0}^{1} \int_{0}^{1} (x^2 + y^4) dy\ dx = \int_{0}^{1} \left(x^2 + \frac{1}{5}\right) = \frac{8}{15}$$

5.2.9

We have that

$$I = \int \int_{R} [f(x)g(y)] dx\ dy = \int_{c}^{d} \int_{a}^{b} [f(x)g(y)] dx\ dy$$

In the integral over x, the term $g(y)$ acts like a constant and hence we can take it out of the integral leaving the following equality

$$I = \int_c^d g(y) \left[\int_a^b g(x)dx \right] dy$$

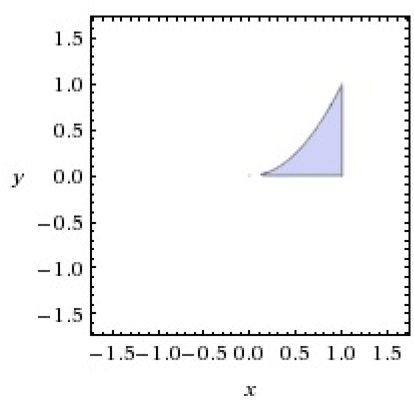
But now all the integral over x is constant on y, and therefore we can take it out of the integral, leaving the desired identity.

5.3.3

(a)

$$\int_0^1 \int_0^{x^2} dy\ dx = \int_0^1 x^2 dx = \frac{1}{3}$$

The region is the following:

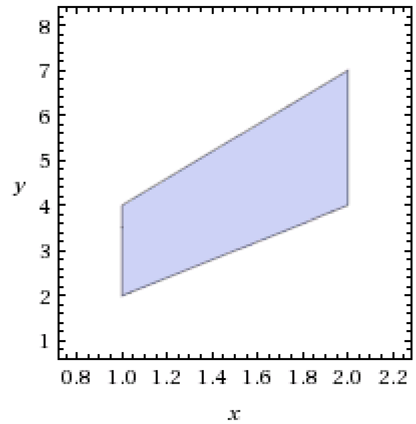


Which is both x and y-simple.

(b)

$$\int_1^2 \int_{2x}^{3x+1} dy\ dx = \int_1^2 (x+1)dx = \frac{5}{2}$$

The region is

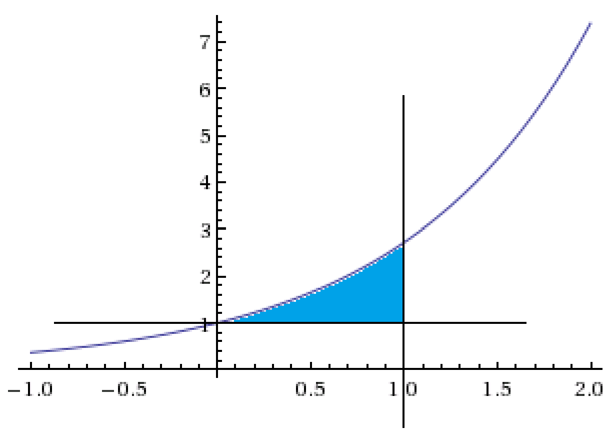


Which is again both x and y-simple.

(c)

$$\int_0^1 \int_1^{e^x} (x+y)dy\ dx = \int_0^1 \left(x(e^x-1) + \frac{e^{2x}-1}{2} \right) dx = \frac{e^2-1}{4}$$

In this case the region is

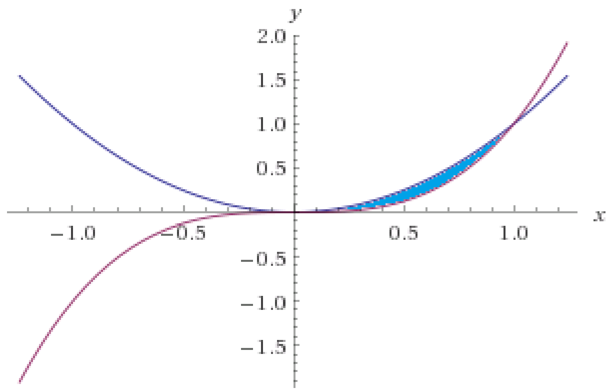


Which is again simple.

(d) Finally

$$\int_0^1 \int_{x^3}^{x^2} dy\ dx = \int_0^1 (x^2 - x^3) dx = \frac{1}{12}$$

Here our region is



Which is simple as well.

5.4.8

The coordinates of the triangle are $(0,0)$, $(10/3, 0)$ and $(0, 5/2)$. This means that x ranges from 0 to 10/3 while y ranges from 0 to $(10-3x)/4$. This gives

$$\int\int_D (x^2+y^2) dA = \int_0^{\frac{10}{3}} \int_0^{\frac{10-3x}{4}} (x^2+y^2) dy\ dx = \int_0^{\frac{10}{3}} x^2 \frac{10-3x}{4} + \frac{(10-3x)^3}{3 \cdot 4^3} dx$$

Which is just the integral of a polynomial, which gives $\frac{5^6}{6^4} \approx 12.056$.

5.4.10

$$\int_0^1 \int_0^{x^2} (x^2 + xy - y^2) dy\ dx = \int_0^1 \left(x^4 + \frac{x^5}{2} - \frac{x^6}{3} \right) dx = \frac{1}{5} + \frac{1}{12} - \frac{1}{21} = \frac{33}{140}$$

Note that the region is the same as in 3) (a).